# Wavelet-based adaptive computation of multiscale flows produced by complex time-dependent geometries: applications to flapping flight.

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We present a novel wavelet-based approach to compute multiscale flows generated by complex, time-dependent geometries. While the framework we present is quite general and can be applied to a large variety of fields, our main motivation for its development comes from the spectacular flight capabilities of flying insects, which use flapping wings to create the required aerodynamic forces. The moving wings require numerical techniques that can track the solution and dynamically adapt the numerical grid accordingly, especially in cases where free flight or deforming wings are considered, since the motion of the boundary is then not known *a priori*, but rather depends on the interaction with the fluid. Here, wavelet-based adaptivity can provide an elegant solution which allows not only to perform simulations previously impossible, but also presents new handles for turbulence modeling. Our framework is inherently based on dynamically evolving grids, which is a requirement that still represents a challenge for an efficient implementation. To this end, we develop a datastructure based on locally regular Cartesian blocks, which are indexed in a tree-like fashion. This datastructure allows, through changing the size of blocks, to balance the interest between a sparse grid and high data regularity. Moreover, the blocks can be distributed among MPI processes and allow an efficient parallelization for large scale supercomputers. To avoid solving elliptic problems, which can be particularly challenging on adaptive nonregular grids, we approximate an incompressible fluid using the method of artificial compressibility, a model equation that introduces a large but finite speed of sound. We thus relax the incompressibility constraint in order to improve the computational efficiency, and, in perspective, to improve error balancing. Since our grid is locally Cartesian, it is by itself not adapted to the geometry of the problem. To this end, we use the volume penalization method, which assumes that both the solid body and the fluid are porous media of different permeability, to include moving and possibly deforming obstacles without the need for a boundary-conformal grid. The combination of artificial compressibility and volume penalization constitutes our physical model, which is well suited for adaptive computations. To solve it numerically, we employ biorthogonal interpolating wavelets as refinement indicators and prediction operators, and combine them with a fourth order finite differences discretization. Using suitable thresholding of wavelet coefficients, we show that the precision of the underlying uniform discretization is maintained on our adaptive grids. The total numerical error of our method can be decomposed into modeling, discretization and thresholding errors, which are all well controlled. For all parameters, we derive scaling relations in order to balance the different error contributions and hence optimize the efficiency of the numerical solution. As our framework is novel, we present detailed validation cases to assess its accuracy, as well as its performance in terms of CPU time and memory compression on massively parallel computer architectures. Finally, we present simulations of a flying model bumblebee and discuss how wavelet thresholding can be used in the spirit of coherent vortex simulation, where a decomposition of the flow into a coherent and a noise-like incoherent part is performed. This decomposition is based on wavelet denoising and presents new perspectives in turbulence modeling. The entire framework is implemented in an open source code which is freely available on the internet[1] in order to maximize its utility for the scientific community.

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[1] https://github.com/adaptive-cfd/WABBIT