Turbulence generated by Fractal Trees

PIV Measurements and Comparison with numerical Data

Master Thesis

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Berlin, 

__________________________
Marc Dreissigacker
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<th>Unit</th>
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<tr>
<td><em>d</em></td>
<td>m</td>
<td>branch diameter</td>
</tr>
<tr>
<td><em>d</em>&lt;sub&gt;0,min,max&lt;/sub&gt;</td>
<td>m</td>
<td>initial/minimum/maximum branch diameter</td>
</tr>
<tr>
<td><em>f</em>&lt;sub&gt;shed&lt;/sub&gt;</td>
<td>Hz</td>
<td>vortex shedding frequency</td>
</tr>
<tr>
<td><em>k</em></td>
<td>m^2/s^2</td>
<td>turbulent kinetic energy</td>
</tr>
<tr>
<td><em>r</em></td>
<td>-</td>
<td>fractional ratio</td>
</tr>
<tr>
<td><em>(u_x,u_y,u_z)</em></td>
<td>m/s</td>
<td>velocity vector in cartesian coordinates</td>
</tr>
<tr>
<td><em>u</em>_&lt;sub&gt;∞&lt;/sub&gt;</td>
<td>m/s</td>
<td>freestream velocity</td>
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<tbody>
<tr>
<td><em>A</em>_&lt;sub&gt;tunnel&lt;/sub&gt;</td>
<td>m^2</td>
<td>windtunnel cross-sectional area</td>
</tr>
<tr>
<td><em>D</em></td>
<td>-</td>
<td>general fractional dimension</td>
</tr>
<tr>
<td><em>D</em>_&lt;sub&gt;ρ,δ&lt;/sub&gt;</td>
<td>-</td>
<td>D regarding length/diameter ratio</td>
</tr>
<tr>
<td><em>J</em>_&lt;sub&gt;max&lt;/sub&gt;</td>
<td>-</td>
<td>number of generations</td>
</tr>
<tr>
<td><em>L</em>_0</td>
<td>m</td>
<td>initial length for the fractal tree generation</td>
</tr>
<tr>
<td><em>N</em></td>
<td>-</td>
<td>number of branches per generation</td>
</tr>
<tr>
<td><em>Re</em></td>
<td>-</td>
<td>REYNOLDS-number</td>
</tr>
<tr>
<td><em>St</em></td>
<td>-</td>
<td>STROUHAL-number</td>
</tr>
<tr>
<td><em>Tu</em></td>
<td>-</td>
<td>turbulence intensity</td>
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### Upper-case Roman
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<td>α</td>
<td>◦</td>
<td>angle</td>
</tr>
<tr>
<td>β</td>
<td>◦</td>
<td>angle</td>
</tr>
<tr>
<td>γ</td>
<td>◦</td>
<td>angle</td>
</tr>
<tr>
<td>δ</td>
<td>-</td>
<td>diameter variation between generations</td>
</tr>
<tr>
<td>θ</td>
<td>◦</td>
<td>angle of rotation around the trunk axis</td>
</tr>
<tr>
<td>ν</td>
<td>m²/s</td>
<td>kinematic viscosity</td>
</tr>
<tr>
<td>ρ</td>
<td>-</td>
<td>length variation between generations</td>
</tr>
<tr>
<td>φ</td>
<td>-</td>
<td>blocking ratio</td>
</tr>
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<tr>
<td>Δt</td>
<td>s</td>
<td>time increment</td>
</tr>
<tr>
<td>Δx</td>
<td>m</td>
<td>particle displacement</td>
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</table>
Abstract

English abstract

The present thesis aims to investigate three dimensional turbulence generators inspired by fractal trees. For this purpose three different multi-scale fractal trees are designed and 3D-printed. In the development process of the structures a focus is set on desired dominant vortex shedding frequencies behind individual cylindrical elements of the tree which should match the wing-beat frequencies of small insects like flies, bumblebees and butterflies in latter experiments. A model wind-tunnel is build and used to conduct PIV-measurements in the $x$-$y$-plane at different heights and distances behind the trees. Their wakes are characterized by mean flow, turbulence intensity and vorticity. Furthermore, measurements with a spruce and a yew-tree are conducted for comparison with real foliage. Furthermore, direct numerical simulations (DNS) are conducted and analyzed for comparison and cross-validation.

Zusammenfassung auf Deutsch

1 Introduction

Nature as the most persistent, iterative optimization algorithm ever to be seen has, over a large number of generations, come up with highly efficient and elegant ways to solve problems. For example, flight has mostly been addressed by development of periodical, flapping motion of flexible wings, as it can be observed with birds and insects. It is to be expected that this approach offers significant advantages, such as high efficiency and probably more advanced control- and adaption-mechanism in order to effectively react to changes in the surrounding area.

Man made flying machines operate with a different approach of employing either rotating or fixed, mostly rigid, wings, as to be observed with helicopters and aircraft. Recent advancements in the technology for flight vehicles, especially in the context of micro-air vehicles (MAVs), require for more advanced methods in order to fulfill today’s requirements. Ideally, this can be achieved by copying and adapting highly optimized methods from nature.

The present work lies within the framework of ‘Aerodynamics of Insect Flight In Turbulent Flow’ (AIFIT), a French-German research project, funded by the Deutsche Forschungsgemeinschaft (DFG) and the Agence Nationale pour la Recherche (ANR). Its overall objective is to improve the understanding of how insects maintain control in highly turbulent environments. Furthermore, advantages of flapping flight in comparison to well-known flight with fixed, rigid wings are to be characterized.

The objective of this thesis is to identify and characterize perturbations these insects are facing in their natural environment, in particular at relatively low airspeeds of $1 - 5$ m/s. In order to do so, tree-inspired turbulence generators are designed in a fractal manner in order to introduce different length- and diameter-scales. In addition, the resulting complex three-dimensional structure can be described by only a few parameters. Desired properties of the tree-generated turbulence are high turbulence intensities and dominant frequencies of the same magnitude as the wing-beat frequency of insects, such
as fruit flies, bumblebees and butterflies. Measurements of the 3D-printed
trees, immersed in laminar flow, are conducted, in a wind tunnel built up for
the present work, applying particle image velocimetry (PIV). Furthermore,
numerical simulations of the same setup are conducted for means of com-
parison. Additionally, measurements with real foliage are conducted.
2 Literature Review: Fractal Tree Turbulence

The literature shows numerous experimental and numerical studies of different fractal grids. Mostly square, I (or H) and square fractal grids are investigated [3, 12, 15, 17, 18]. Only little could be found concerning fractal trees as turbulence generators [5, 29], even less three-dimensional structures [1, 4]). Furthermore, studies concerned with turbulence within real plant canopy [10, 25, 34, 38] are considered.

Van Hout et al. carried out PIV measurements in the field combined with the usage of meteorological sensors within and above a mature corn canopy, while their measurements don’t support Finnigan’s proposition of a spectral shortcut [10] but rather match the classical -5/3-slope in the spectrum. PIV-data of Zhu et al. [38] suggest a similar behavior within the canopy, while larger deviations from the -5/3-slope are found for smaller wavenumbers.

Furthermore, Van Hout et al. found a strong correlation between the dissipation rate and the out-of-plane component of the vorticity, even though their measurements are too coarse for calculating the dissipation rate from velocity gradients or to calculate the vorticity with sufficient accuracy.

Poggi et al. [25] examined canopy sub-layer turbulence with Laser-Doppler Anemometry for different canopy densities. For their experiments, they investigated arrays of vertical steel cylinders with 0.12 m in height and 4 mm in diameter with Reynolds-numbers of around 175000. Their findings suggest that, given the connection between mean flow and vortex shedding frequency (Strouhal’s relation), dominant frequencies in the spectrum within the trees exist. They furthermore argue that these peaks are connected to energy short-circuiting to scales of around the small wakes behind the thin steel rods. Additionally, they find the mixing length being linked to the vortex diameters. They derive an intermediate model covering the range between no and full presence of canopy. Without strong presence of canopy, the behavior resembles a rough sub layer. In the other case, strong presence of canopy,
mixing-layer behavior is found. They show that the dominant mechanism of the internal flow field is the von-Kármán vortex street, which is periodically perturbed by sweep events which, themselves, are influenced by the canopy density.

In their paper Hurst and Vassilicos [15] examine 21 different planar fractal grids, cross-, I- and square-fractal grids, in two different wind tunnels with hot-wire anemometry. They find a strong dependence of homogeneity, isotropy and decay properties from the fractal dimension, which is <2 for all their grids. Their grids have blocking ratios of up to 25%. They find that the turbulence peaks after some distance behind the grid and then decays in an exponential manner. This contradicts the classical description of (non-fractal-) grid turbulence, which is done using a power law [26]. Their findings might not be applicable to the present study, since now the fractal turbulence generators are three-dimensional and therefore peaking at one certain distance is not expected.

In their paper Cafiero et al [3] examine the near wake of jets with fractal cross-, I- and square-fractal grids with PIV in a water tunnel at a Reynolds-number of 28000. They analyze the coherent structures and, via Proper Orthogonal Decomposition, find that about 70% of the modal energy can be linked to the vortex shedding from the largest bars. Gomes et al. [12] experimentally investigated space-filling planar fractal square grids using PIV in water. They introduce a novel generalization for the wake-interaction length scale with a turbulence intensity scaling which, together, collapse the data for several measurements with different grids. It includes free-stream turbulence, as well as grid geometry and therefore is an extension to the findings of Hurst and Vassilicos.

Another study of fractal-generated turbulence was carried out numerically by Laizet and Vassilicos [18]. They perform Direct Numerical Simulations for one regular grid and three different fractal square grids with a Reynolds-number of 300 at the smallest diameter, which corresponds to a free-stream velocity of 2.5 m/s. Their results suggest that the vorticity field is more clustered when generated by fractal grids, as well as a higher vorticity and turbulence intensity for fractal square grids in general. They highlight the geometrical imprint of the fractal structure far downstream. Again, they dis-
tistinguish between a production region close behind the grid and the decay region located further downstream. Another study of was carried out by Laizet and Vassilicos [17] in their paper *Multiscale Generation of Turbulence*, which offers a great introduction into fractals and their use in science. They also highlight the very limited market penetration of concepts and products employing fractals. Their study consists of Direct Numerical Simulations with the Immersed Boundary Method and suggests that RANS and also LES models cannot be used to model fractal-generated turbulence because it differs from know forms of turbulence in fundamental ways. Mostly, the TKE-Dissipation rate depends on the Reynolds-number. The Kolmogorov phenomenology as underlying principle for LES sub-grid modeling and RANS-models assumes independence from the Reynolds-number. Their results still match the previously discussed studies on fractal cross-, square and I-grids, while they highlight the dependence on the Reynolds-number.

Two dimensional fractal trees were examined by Chester et al [4, 5] and Schröttle [29]. Chester et al. considered high-Re flows only with a focus on the drag-forces for both 2D-trees and also a 3D-Tripod-tree. Their findings include that the drag coefficient significantly increases with the fractal dimension. They performed Renormalized Numerical Simulations, making use of the trees self-similar scales in order to reduce computational cost. In [29] 16 fractal Pythagoras trees are modeled as immersed boundaries within a Large Eddy Simulation, which resolves the vortex shedding behind individual branches up to scales of the atmospheric surface layer. Their focused on thermal effects, while the tree-crowns were set to be 3.35 K warmer than the ambient air. They show that the TKE power spectra follow the $-5/3$ slope for the trunk, crown. For large wavenumbers the internal boundary layer above the canopy massively loses energy, which the authors see as an indicator for energy being injected into the flow at a characteristic scale of the trees from wakes behind their respective branches.

In his dissertation Bai [1] thoroughly investigated turbulence behind a three-dimensional fractal tree, as well as for a large array of them, for a Reynolds-number of 71300 in water using PIV. With matching of refractive indices
and invoking translucent trees, three branches per each of the four generations, measurements inside the canopy were carried out. Mean velocity, Reynolds-Stresses and the downstream development are discussed. Structures of the tree can be found in the near-wake velocity and turbulence profiles. A measured mixing length study indicates good agreement with the Boussinesq eddy-viscosity principle. Still, the variety of length scales is challenging for modeling mixing lengths and should therefore, according to Bai, be incorporated into the model.
### 3 Fractal Trees

In the present work turbulence generators inspired by fractal trees are thought up, employed, both experimentally and in simulations, and discussed. The key benefit of this approach is the possibility to fully describe the structures with only few parameters which are shown in table 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_0$</td>
<td>some arbitrary diameter to start with</td>
</tr>
<tr>
<td>$L_0$</td>
<td>some arbitrary length to start with</td>
</tr>
<tr>
<td>$N$</td>
<td>number of branches per generation</td>
</tr>
<tr>
<td>$J_{\max}$</td>
<td>number of generations</td>
</tr>
<tr>
<td>$\rho$</td>
<td>variation of length between consecutive generations</td>
</tr>
<tr>
<td>$\delta$</td>
<td>variation of diameter between consecutive generations</td>
</tr>
<tr>
<td>$[\alpha, \beta, \gamma]$</td>
<td>Euler angles defining orientation of each branch in the reference system of the parent structure</td>
</tr>
</tbody>
</table>

Table 3.1: Parameters describing general fractal trees

The generation of such a self-similar tree is done recursively:
First, the trunk, which is not part of the fractal structure yet, is created. From its tip, the first generation ($J = 1$) of branches is created. At the tip of each branch, the next generation of branches emerges, using the local coordinate system of its parent branch. This procedure repeats until the desired level of generations, $J_{\max}$, is reached. The tree can then be seen as a superposition of the trunk and its branches.

Furthermore, an important index providing information about self-similar structures was introduced by Mandelbrot in 1967, the fractional dimension [19, 20]. He defines it as follows:

$$D = -\frac{\log(N)}{\log(r(N))}. \quad (3.0.1)$$

Applying rules for computing logarithms and taking into account that $r$, the fractional ratio, is independent of the dimensions, thus constant, eq. 3.0.1
24 Fractal Trees

reads

\[ D = \frac{\log(N)}{\log(\frac{1}{r})}. \]  

(3.0.2)

The present trees are three-dimensional structures with a constant ratio between consecutive generations regarding length as well as diameter. Therefore, two fractional dimensions can be defined,

\[ D_\rho = \frac{\log(N)}{\log(\frac{1}{\rho})}, \]  

(3.0.3)

\[ D_\delta = \frac{\log(N)}{\log(\frac{1}{\delta})}, \]  

(3.0.4)

where \( D_\rho \) denotes the fractional dimension regarding the length ratio \( \rho \), whereas \( D_\delta \) is calculated using the diameter ratio \( \delta \). As suggested in [37], \( 2 < D < 3 \) is valid for a large variety of tree foliage. This is included in the design of the trees, compare table 3.2.

\section*{3.1 Design & Choice of Parameters}

The following section is intended to discuss the choice of parameters for the different trees. For the purpose of finding suitable trees, different combinations of parameters are investigated. Properties such as a space-filling structure or the presence of a variety of different length- and diameter-scales are desired. Furthermore, the branches must not be too far apart from each other in order for their wakes to interact and thus generating turbulence more effectively. The tree is assumed to be rigid while leaves are neglected. At this point it remains unclear whether the spectral shortcut, as suggested by FINNIGIN in [10] applies without the foliage but only with the branches.
3.1.1 First Approach in MATLAB

A script in MATLAB is used to construe the trees and to get a basic idea about their appearance. It is used for a first, yet rough, estimate of vortex shedding frequencies given the simplifying assumption of no interaction between the branches’ wakes, i.e. neglecting all nonlinear behavior. The script yields a visualization of the tree (compare figures 3.1.1 - 3.1.3) and an ASCII-file containing all points, $x_{\text{base}} = (x_b, y_b, z_b)$ and $x_{\text{tip}} = (x_t, y_t, z_t)$ for each branch, as well as the corresponding radii. The idea of the script, however, is to generate the tree by simply sticking the cylindrical branches’ centerlines together. Intersections between cylinders at tip-end connections are not present yet, since they are only lines.

Three sets of parameters turned out to match the desired criteria to an adequate degree. These sets, specific trees, were decided to get examined more thoroughly. The first tree, the H-tree, is designed to be space-filling, yet it only vaguely resembles a tree. However, it consists of nine different branch-lengths and -diameters and offers small distances between branches. It is inspired by the two-dimensional, area-filling, tree presented in [22] and by its tree-dimensional versions, also shown in [22], [36] and [35]. The second tree, called Pyramid-tree, appears to look much more like a tree, in particular a coniferous tree. Still, it resembles a pyramid, which gave it its name. It can again be characterized as space filling and consists of five generations. Its two-dimensional representation was found in [5] and [22]. The third structure, the Spherical tree, consists of branches whose tips, for each generation, seem to create a roughly spherical shape. Its general appearance was inspired by figures presented in [22] and [35]. In contrast to the two other trees, its internal structure is not space-filling. Yet, and again in contrast to the previous structures, the branches have inclination angles which differ from $90^\circ$. Within each of the seven distinct spherical areas, the distance between branches is smaller than in between these spheres. An isometric view, as well as an orthographic view of each tree can be found in figures 3.1.1 - 3.1.3. Especially the orthographic views clarify the naming
and show the area-, respectively space-filling properties of both the \emph{H-} and the \emph{Pyramid-tree}. Furthermore, the evolution of the H-TREE is presented in figure 3.1.4.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fractal_trees}
\caption{H-tree created in MATLAB}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fractal_trees2}
\caption{Pyramid-tree created in MATLAB}
\end{figure}
Fractal Trees

(a) Isometric view   (b) Orthographic view

Figure 3.1.3: Spherical tree created in MATLAB

Figure 3.1.4: Evolution of the generation of the H-tree from $J = 1 - 9$
3.1.2 Vortex Shedding Frequency and Branch Diameters

The maximum and minimum branch diameters are derived using Strouhal’s relation. It connects vortex shedding frequency $f_{\text{shed}}$ behind a cylindrical branch to its diameter $d$, the free-stream velocity $u_\infty$ and the Strouhal-number. Written in the form which is used here, it reads

$$d = \frac{St \cdot u_\infty}{f_{\text{shed}}}.$$  \hspace{1cm} (3.1.1)

Given the free-stream velocity of 1 - 4 m/s, which can be realistically achieved with the experimental setup available, maximum and minimum values for the local Reynolds-number can be derived, which is commonly defined as

$$Re = \frac{u_\infty \cdot d}{\nu}.$$  \hspace{1cm} (3.1.2)

In order to achieve maximum perturbations for the insects, the vortex shedding frequencies are desired to match their wing-beat frequencies. As stated in [6], the average wing-beat frequency for butterflies is at around 11 Hz. Bumblebees, according to [7], have an average wing-beat frequency of 150 Hz, while [21] indicates a value of 218 Hz for fruit flies. Hence, shedding frequencies in the range of 10 - 250 Hz are desired.

For reasons of experimental feasibility and dimensions of the wind-tunnel, the maximum diameter is determined to $d_{\text{max}} = 0.02$ m, which applies for the trunk and the first generation. The minimum diameter is $d_{\text{min}} = 0.0015$ m and applies for the last generation, respectively the finest branches. The lower limit for the branch diameter is chosen according to the limitations of the 3D-printing process. For air ($\nu = 15 \cdot 10^{-6} \text{m}^2/\text{s}$) equation 3.1.2 yields $Re_{\text{max}} = 5333$ (for $u_\infty = 4 \text{m/s}$ and $d = 2 \text{cm}$) and $Re_{\text{min}} = 100$ (for $u_\infty = 1 \text{m/s}$ and $d = 1.5 \text{mm}$).

According to [2], within the present range of Reynolds-numbers, periodical vortex shedding occurs. Since the St-number depends on Re, results from [9] are used to include this dependency into the estimate. This yields $St_{\text{max}} = 0.208$ and $St_{\text{min}} = 0.165$ with an average $St_{\text{mean}} = 0.186$, which, for
simplicity, is used for further calculations. Again, combinations of maximum and minimum values for $u_\infty$ and $d$ are used to calculate the frequencies. It yields $f_{shed,max} = 496.8$ Hz and $f_{shed,min} = 9.3$ Hz. The range of shedding frequencies, hereby still completely neglecting nonlinear behavior, includes the insects’ wing-beat frequencies and therefore seem to be suitable for the task of maximum perturbations for the insects’ flight.

The maximum and minimum branch diameters are used to calculate the diameter ratio $\delta$ using the relation

$$\delta = \frac{d_{min}^{1/l_{\max}-1}}{d_{max}}.$$  \hspace{1cm} (3.1.3)

Table 3.2 summarizes the relevant parameters for the discussed trees, including the complete set of angles, the dimensions in x-, y- and z-direction, as well as length and diameter ratios. Furthermore, the overall number of cylinders forming the tree and the number of finest branches, $N_{l_{\max}}$, are presented. The branch is rotated applying rotation matrices in the following manner, where $R$ denotes the rotation matrix around the respective axis and $M$ is the rotation matrix of a branch: $M_{out} = R_x(\gamma) \cdot R_y(\alpha) \cdot R_z(\beta) \cdot M_{in}$. Details about the script generating the trees can be found in the appendix.
<table>
<thead>
<tr>
<th>J_{max}</th>
<th>H-tree</th>
<th>Pyramid-tree</th>
<th>Spherical tree</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>9</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>L_{trunk}</td>
<td>24 cm</td>
<td>22.7 cm</td>
<td>16.3 cm</td>
</tr>
<tr>
<td>L_0</td>
<td>12 cm</td>
<td>11.35 cm</td>
<td>14.2 cm</td>
</tr>
<tr>
<td>ρ</td>
<td>2^{-\frac{1}{3}}</td>
<td>\frac{1}{2}</td>
<td>\frac{2}{5}</td>
</tr>
<tr>
<td>δ</td>
<td>0.723</td>
<td>0.523</td>
<td>0.422</td>
</tr>
<tr>
<td>D_ρ</td>
<td>3</td>
<td>2.322</td>
<td>2.124</td>
</tr>
<tr>
<td>D_δ</td>
<td>2.141</td>
<td>2.485</td>
<td>2.254</td>
</tr>
<tr>
<td>α [°]</td>
<td>[90 90]</td>
<td>[0 90 90 90 90]</td>
<td>[0 65 65 65 90 90 90]</td>
</tr>
<tr>
<td>β [°]</td>
<td>[0 180]</td>
<td>[0 0 90 180 270]</td>
<td>[30 0 120 240 60 180 300]</td>
</tr>
<tr>
<td>γ [°]</td>
<td>[90 90]</td>
<td>[0 0 0 0 0]</td>
<td>[0 0 0 0 0 0]</td>
</tr>
<tr>
<td>width dx</td>
<td>37.2 cm</td>
<td>44.7 cm</td>
<td>39.4 cm</td>
</tr>
<tr>
<td>depth dy</td>
<td>33.3 cm</td>
<td>44 cm</td>
<td>44 cm</td>
</tr>
<tr>
<td>height dz</td>
<td>42 cm</td>
<td>44 cm</td>
<td>39.9 cm</td>
</tr>
<tr>
<td>Σ(cylinders)_{total}</td>
<td>1023</td>
<td>3902</td>
<td>2801</td>
</tr>
<tr>
<td>Σ(cylinders)<em>{J</em>{max}}</td>
<td>512</td>
<td>3125</td>
<td>2401</td>
</tr>
</tbody>
</table>

Table 3.2: Relevant parameters for the trees

3.1.3 Generation of 3D-STL-mask with FLUSI

The ASCII-file created by the MATLAB-script can be read by FLUSI, the fluid-structure interaction code developed by Thomas Engels [8]. It generates cylinders from lines with the given centerlines and radii, sets hemispheres onto the blunt ends of each cylinder and superimposes them. It also creates a smoothing layer, such that the mask function χ(x, y, z) is 0 outside the tree domain and 1 inside of it. In between, the smoothing layer ensures smooth transition, which is beneficial for numerics [16]. An iso-surface of the mask-function at χ = 0.5 yields the surface of the tree.

The surface data obtained from the mask generation is presented in figures 3.1.5 - 3.1.7.
Figure 3.1.5: H-tree created with FLUSI

(a) Isometric view  
(b) Orthographic view

Figure 3.1.6: Pyramid-tree created with FLUSI

(a) Isometric view  
(b) Orthographic view
Figure 3.1.7: Spherical tree created with FLUSI
3.2 3D-printing and Assembly

This section is intended to discuss the preparations, execution and results of the 3D-printing process. Since each tree is larger than the printing chamber, the models have to be subdivided into smaller pieces in order to ensure printability. The trees’ fractal properties allow dividing the tree into few different parts, which can each be printed multiple times.

3.2.1 Preparation in SOLIDWORKS

The STL-surface mesh from FLUSI can easily be divided into several sub-meshes, but since it is only points and vertices without underlying (exact) shapes or constraints, manipulation in a controlled manner is found to be work-intensive. Especially since pin-hole connections between the parts of the trees are desired. In order to bypass the inconvenient process of modeling solid parts onto surface data, the whole structure is modeled in SOLIDWORKS. Again, the fractal properties of the structures are used by applying mirroring and patterns. As stated in section 3.1.3, each trunk and first generation of branches has a diameter $d_{\text{max}} = 2\text{ cm}$. These parts are, for saving cost and time, made from wooden rods. The models are shown in figures 3.2.1 - 3.2.3. Wooden parts are indicated with brown color.

![Isometric view](image1.jpg) ![Orthographic view](image2.jpg)

Figure 3.2.1: H-tree created with SOLIDWORKS
Figure 3.2.2: Pyramid-tree created with SOLIDWORKS

Figure 3.2.3: Spherical tree created with SOLIDWORKS
In addition to brown wooden rods, identical parts are given the same color. In 3.2.3 it is clearly visible that besides the wooden rods, there are only three different parts to be printed: The connector between the trunk and first generation of branches (gray) and seven connectors between the first generation and the smaller structures (green). The 49 smaller structures, which include the second, third and fourth generation, complete the tree (red). The other trees follow the same underlying principle. In table 3.3 the number of components which have to be assembled in order to create the tree can be seen.

<table>
<thead>
<tr>
<th></th>
<th>H-tree</th>
<th>Pyramid-tree</th>
<th>Spherical tree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>2</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>3D-print</td>
<td>191</td>
<td>31</td>
<td>57</td>
</tr>
<tr>
<td>Total</td>
<td>193</td>
<td>37</td>
<td>65</td>
</tr>
</tbody>
</table>

*Table 3.3: Number of components of the trees*

### 3.2.2 Printing the H-tree

The components for the *H-tree* are printed using a *Formlabs Form 1+* 3D-printer. Detailed information about axis resolutions, build volume dimensions, laser specifications and the software *PreForm* can be found in [27]. Figure 3.2.4 exemplarily describes the production stages of a printed component. In figures 3.2.4b and 3.2.4c the support structures created during the process, in order to prevent the components from bending or even breaking, are shown. The surface of a printed component is displayed in more detail in figure 3.2.5, where the different layers are clearly visible.
Fractal Trees

Figure 3.2.4: From the 3D-model to the printed component

(a) 3D-model (SOLIDWORKS)
(b) Model with supports (PREFORM printing software)
(c) Print with support structures
(d) Print ready for assembly

Figure 3.2.5: Surface structure of a printed component
Unfortunately, the accuracy, especially the preservation of angles (90° and 180°) in the pin-hole connectors, was not satisfactory. Hence, there is a considerable discrepancy between the resulting tree and the exact model. The deviations add up because the structure consists of a large number of printed parts (191). Since a good accordance between the printed tree for experiments and the geometry for simulations is needed to make meaningful comparisons, no measurements with the H-tree are conducted. In figure 3.2.6 the inaccuracy is clearly evident.

![Figure 3.2.6: The printed and assembled H-tree](image-url)
3.2.3 Printing the Pyramid- & Spherical Tree

In contrast to the *H-tree*, the *Pyramid- and spherical tree* are printed using *EOS Formiga P100* with a significantly larger building volume and better results in terms of accuracy. For that reason, as shown in table 3.3, less, but larger components are needed to construct the trees. Densely packed building volumes, arranged in *MATERIALISE MAGICS 19.01*, are presented in figure 3.2.7. The assembled trees are shown in figures 3.2.8 and 3.2.9.

*Figure 3.2.7: Densely packed building volumes*

*Figure 3.2.8: The printed and assembled Pyramid-tree*
Additionally, one of the seven large branches of the *Spherical tree* is used for PIV measurements in order to obtain more detailed information about its wake. It then can be determined if and how strong interactions between the larger branches’ occur. This branch consists, like the original tree, of all branch diameters. Its height is 21.2 cm whilst being 17.1 cm wide and 15.5 cm in depth. The former first generation branch here is the trunk. They both have the same diameter of 2 cm. The 3D-CAD model and the branch mounted in the wind-tunnel can be seen in figure 3.2.10. In the background the turbulence sieves and the fan-array are visible.

Conducting a Direct Numerical Simulation (DNS) of a whole tree with a reasonable resolution, even for the smallest branches, exceeds the limits of current computational power. For this reason measurements and simulations for very small parts of the trees are executed. Given their fractal character, these parts are smaller versions, obviously with less branches, of the trees themselves. Investigations are conducted for the *Pyramid* - and *Spherical tree*. As shown in figure 3.2.11a, the small *Pyramid-tree* consists of branches from the second, third, fourth and fifth generation. It has a total height of 8.7 cm and a maximum width and depth of 10.1 cm. In the same figure on the right, 3.2.11b the small *Spherical tree* is shown, which consists of second, third and fourth generation branches with a total height of 6.9 cm, width of 5.7 cm and depth of 6.2 cm.
(a) Branch created with SOLIDWORKS  
(b) Branch installed in the windtunnel

Figure 3.2.10: The small trees for more detailed studies

(a) Small branch of the Pyramid-tree  
(b) Small branch of the Spherical Tree

Figure 3.2.11: The small trees for more detailed studies
4 Experimental Setup & Validation

This chapter is intended to illustrate the experimental setup. The wind-tunnel and the PIV system are discussed, the MATLAB-toolkit PIVLAB, which is used for analysis of the data, is presented and validation cases are documented.

4.1 Windtunnel

The experiments are carried out in the Technische Akustik Prüfalle on the campus of Technische Universität Berlin. The wind tunnel, built for conducting the experiments with the fractal tree turbulence generators, is made from acrylic glass with a thickness of 3 mm and wooden frames to stabilize the structure. Figure 4.1.1 shows a model of the tunnel with pieces cut out of the fan-array, honeycombs and two turbulence sieves for demonstration purposes. The tunnel is designed with a removable top for easy installation of the different trees into the test section.

The measuring section, starting after the second turbulence sieve, is rectangular with a width of 0.474 m and height of 0.477 m ($A_{testsection} = 0.23 \text{ m}^2$). It is 1.7 m long, while 1.5 m of it, in between the two wooden frames, has optical access for the PIV measurements. The trees are placed through a 20 mm hole in the bottom, which is located 0.5 m downstream from the beginning of the visible measuring section. The coordinate system has its point of origin on the tunnel bottom in the center of the trunk.

The tunnel is placed in a rack built with aluminum-beams, which also contain laser-unit and camera. The setup is shown below in figure 4.1.2.
Figure 4.1.1: Model of the windtunnel

Figure 4.1.2: The experimental setup
4.1.1 Blockage

The area blockage ratio $\phi$ is determined by calculating the projected area of the model in direction of $u_\infty$ divided by the cross-sectional area of the tunnel. $\phi$ should, as proposed in [30], not exceed 10%. In [14] a value for $\phi < 20\%$ is suggested.

$$\phi = \frac{A_{\text{projected}}}{A_{\text{testsection}}} \quad (4.1.1)$$

Table 4.1 provides the results, the data for $\theta = 0^\circ$ applies for orientations like presented in figures 3.2.1b, 3.2.2b and 3.2.3b. Rotations are realized around the trunks’ center axis. For simplicity, the ratios were only determined for angles which coincide with symmetry planes. The numbers exceed 10%. However, the trees’ blockage is not focused to a distinct part of the cross sectional area, it is distributed over most of it. This leads to the assumption that exceeding the threshold of 10% is justified. Local accelerations around the fine branches do not lead to the obstacle acting as a nozzle, like it would be the case for a solid body, e.g. a car-like model.

<table>
<thead>
<tr>
<th>H-tree</th>
<th>Pyramid-tree</th>
<th>Spherical tree</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_H$</td>
<td>$\phi_H$</td>
<td>$\theta_{py}$</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>9.88 %</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>7.89 %</td>
<td>$45^\circ$</td>
</tr>
</tbody>
</table>

Table 4.1: Blocking ratio $\phi$ for each tree at different angles $\theta$
4.2 PIV System

Particle Image Velocimetry (PIV) is a non-intrusive, image-based method for measuring velocity fields. The principle is presented in figure 4.2.1, which has been adapted from [28].

![Figure 4.2.1: The principle of PIV, adapted from [28]](image)

The flow is seeded with tracer particles. A laser with light sheet optics illuminates a plane twice in a short time. The laser-pulses are synchronized with the camera, which takes one picture of the illuminated particles for each pulse with a known time increment \( \Delta t \). The two images are then divided into smaller interrogation areas, of which the correspondent areas are cross-correlated, which, if done properly, yield the direction and distance of the displacement, \( \Delta x \). The relation

\[
u = \frac{\Delta x}{\Delta t}
\]

holds for sufficiently small values of \( \Delta t \), because movement with local flow speed in local flow direction can with a fair degree of certainty assumed to
be linear. Hence, one velocity vector per pair of interrogation window can be derived. Details about the flow-following abilities of the seeding particles, their light-scattering properties, statistical methods, as well as multi-pass techniques and several filtering approaches can be found in [23] and especially in [28]. The inherent accuracy limit, which is of order 0.1 px, is described in [24].

In the experiments for this work, seeding is generated from DEHS-oil with an atomizing nozzle, which produces oil droplets of around 1 $\mu m$ in diameter. The images are recorded with an ILA pco.2000 cooled 14-bit CCD-Camera with a maximum resolution of 2048 x 2048 pixels while operating at 15 Hz. The exposure time for each image was set to 20 ms. For illumination, a Quantel EverGreen 145 double-pulse Nd:YAG-Laser with a wavelength of 523 nm and a maximum power of 145 mJ per pulse is used. A pulse distance of 293 $\mu$s was chosen for the double images, such that the displacement of tracer particles in a flow of 3 m/s is around 8 px. Some examples of recorded images and the resulting vectors, including valid and invalid ones, can be found in the appendix.
4.3 Analysis Software PIVLAB

For derivation of the velocity field, which is the basis for all further analyses, PIVLAB - TIME-RESOLVED DIGITAL PARTICLE IMAGE VELOCIMETRY TOOL FOR MATLAB is used, an open-source GUI based tool. It was first published in March 2009 and has been updated and improved ever since. The analyses made for the present work were done using version 1.41 [32]. It contains features such as masks, the option to define a range of interest (ROI) and different pre-processing image enhancement options, e.g. high-pass filtering or intensity capping for overly bright pixels. Furthermore, it offers up to four analysis-passes with different sized interrogation areas for each pass, as well as different sub-pixel estimators. Vector-validation can be achieved with median or standard-deviation filters and as well by validation based on a u-v-scatter-plot. PIVLAB offers various means of extracting data from the calculated velocity field, such as extraction of velocity profiles along arbitrary lines, integration of variables in any area, as well as subtracting mean flows, histograms, calculation of the vorticity, streamlines or shear and strain rates. The obtained datasets can be exported in different formats, such as mat or vtk, of which the later is used for further analysis in PARAVIEW. Detailed descriptions and documentation about PIVLAB can be found in [31] and [33], while images of the settings used in the present work can be found in the appendix.
4.4 Validation Cases

In order to validate the recorded data and analysis routine, measurements of both the empty canal with and without flow are conducted at two different positions. Hence velocities around 0 m/s and $u_\infty$, within the whole system’s accuracy, are measured. The results then are used to assess the accuracy of measurement and to derive information about the undisturbed velocity field and its initial fluctuations. According to the coordinate system introduced in figure 4.1.1, the corresponding velocity components $u_x$, $u_y$, and $u_z$ can be decomposed into a stationary (mean) velocity $\overline{u}$ with instationary fluctuations $u'$ around the mean value. Furthermore $\overline{u}$ is calculated by temporal averaging of measured velocities for $\Delta t = const$. Since the fluctuations describe variations around a mean value, their temporal average is zero (equation 4.4.3). Hence, the second-order statistical moment, the RMS value (root mean square, equation 4.4.4), is usually used to characterize fluctuations [11, 23, 26].

\[
\begin{align*}
  u(x,t) &= \overline{u}(x) + u'(x,t) \\
  \overline{u}(x) &= \frac{1}{n} \sum_{i=1}^{n} u(x,t_i) \\
  u'(x,t) &= 0 \\
  \sqrt{\overline{u'^2}} &= \sqrt{\frac{1}{n} \sum_{i=1}^{n} u'(x,t_i)^2}
\end{align*}
\]

In addition the mean turbulent kinetic energy $k$ can be calculated. In the case of isotropic turbulence, the statistical properties of the velocity are invariant with respect to translation and rotation, which simplifies the calculations and offers an estimate for $k$ with only 2D-data, although fluctuations, and therefore $k$, are always three-dimensional. The last component $\overline{u_z'^2}$ is, because of assuming isotropic turbulence (equation 4.4.6), replaced by the sum of each 1/2 of the two other components [11, 13, 26].
k = \frac{1}{2}(\overline{u_x'^2} + \overline{u_y'^2} + \overline{u_z'^2}) \quad (4.4.5)

\overline{u_z'^2} = \frac{1}{2}(\overline{u_x'^2} + \overline{u_y'^2}) \quad (4.4.6)

k_{2D} = \frac{3}{4}(\overline{u_x'^2} + \overline{u_y'^2}) \quad (4.4.7)

The turbulence intensity, especially for the measured 2D data is calculated as follows. Equation 4.4.9 will be used for calculations.

\begin{align*}
Tu_{3D} &= \sqrt{\frac{1}{3}(\overline{u_x'^2} + \overline{u_y'^2} + \overline{u_z'^2})} = \frac{\sqrt{2/3}k}{\bar{u}} \\
Tu_{2D} &= \sqrt{\frac{2}{3}k_{2D}} = \sqrt{\frac{1}{2}(\overline{u_x'^2} + \overline{u_y'^2})} = \frac{\sqrt{1/2}(\overline{u_x'^2} + \overline{u_y'^2})}{\bar{u}}
\end{align*} 

(4.4.8) (4.4.9)

The configuration of the PIV-system and measured validation planes is presented in figure 4.4.1. The plane closer to the tree, validation plane 1, is a square with an edge length of 23.3 cm, which is centered at (23 cm, 11.5 cm, 11.5 cm). Validation plane 2 has dimensions of 23.3 cm x 20.5 cm with its center located at (48.7 cm, −12 cm, 6.5 cm).

Figure 4.4.1: Orientation of the planes for the validation case
The analysis is performed applying three passes, each with 50% overlap. The first one uses window sizes of 64 x 64 pixels, the second operates with 32 x 32 and the third pass then works with the final interrogation area of 16 x 16. Furthermore, a Gauss 2 x 3 point sub-pixel estimator is applied. The FFT window deformation algorithm with a spline window deformation interpolator is chosen. For each pass, the corresponding displaced and deformed interrogation window, obtained from the previous pass, is used for correlation. This reduces effects, such as peak-locking, since with the deformed windows are displaced in direction of the local velocity obtained from the previous pass. That reduces the amount of particles having left the interrogation windows within $\Delta t$.

In the following analyses, poorly illuminated and reflecting areas, from the aluminum frame under the wind tunnel, are clipped. Histogram equalization (CLAHE), high-pass filtering in order to remove structures in the background of the images, as well as intensity capping are applied. Furthermore, the resulting vectors are filtered by standard deviation (default settings) and by discarding erroneous vectors using the scatter-plot. For instance, vectors represented by dots outside the dense cluster in figure 4.4.2d are discarded and replaced by interpolated vectors.

Different interrogation window sizes have been tested. In order to achieve the highest resolution, the windows are chosen to be as small as possible. Scatter-plots of the measured velocities in validation plane 2 are presented in figure 4.4.2 for interrogation window sizes of 8,16 and 32 px for configurations without flow on the left and with flow on the right. It is visible that for 8 px the amount of erroneous results increases in comparison to larger sizes. On the other hand further increasing the windows to 32 px yields reduces the spatial resolution even more but there seems to be no benefit in terms of the accuracy of the results. This is deduced from the fact, that the scatter-plots for both configurations appear to be very similar for both 16 and 32 px. Therefore 16 px windows are used for all further analyses. For an image edge length of 23.3 cm at 2048 pixels, the analysis yields 255 vectors. Hence, a 255 x 255 vector field with each vector describing an area of 0.91 mm x 0.91 mm is obtained. For plane 1 300 samples are recorded, for
plane 2 only 75. The therefore better convergence of the turbulence statistics in plane 1 can be seen in figures 4.4.5 and 4.4.8.

Figure 4.4.2: Velocity scatter-plots, empty wind-tunnel, with and without flow, different interrogation windows
The velocity magnitude in validation plane 2 for the empty wind tunnel without flow can be seen in figure 4.4.4. Note the increased noise for the 8 px window in 4.4.4a. The vertical drift in the results, between 0 m/s and about 0.025 m/s, seems to be independent of the window size. For validation plane 1 values within the limits ±0.02 m/s were found. The experimental setup is able to measure a velocity of zero within limits of ±0.05 m/s.

The velocity magnitude for validation plane 1, calculated with a window size of 16 px, is presented in 4.4.3. It is worth noting that the velocity magnitude only in small areas exceeds 0.15 m/s, which indicates an even better accuracy than achieved in validation plane 2. Furthermore, it should also be noted that structures, like parts of the aluminum (see 4.1.2), from the image background are visible. As mentioned above, poorly illuminated or reflecting parts of the images are clipped. Here this is not done for demonstration purposes. Clearly, the measurements are distorted. The results obtained still seem to be reasonable and therefore is would be misleading to include them into the analysis. Still, the non-distorted area indicates values being closer to a velocity of 0 m/s than the values obtained from validation plane 2.

Figure 4.4.3: Velocity in the empty channel, no flow, 16 px, validation plane 1
Experimental Setup & Validation

(a) Interrogation window size 8 px, without flow

(b) Interrogation window size 16 px, with flow

(c) Interrogation window size 32 px, without flow

Figure 4.4.4: Velocity in the empty channel, no flow, different interrogation windows, validation plane 2
The same procedure is repeated with the tunnel running. For the velocity magnitudes, see figure 4.4.5. The histograms for both validation planes can be found in figures 4.4.6 and 4.4.6. The average rounded to the first decimal yields $\bar{u} = 2.8$ m/s, which will be used for further calculations.

![Velocity magnitudes for the empty channel with flow, 16 px](image1)

(a) Validation plane 1  
(b) Validation plane 2

*Figure 4.4.5: Velocity magnitudes for the empty channel with flow, 16 px*

![Histogram for the empty channel with flow, 16 px, validation plane 1](image2)

*Figure 4.4.6: Histogram for the empty channel with flow, 16 px, validation plane 1*
The turbulence intensities can now be calculated via 4.4.9. The results can be found in figure 4.4.8. It can be seen that the turbulence intensity for the validation planes, and hence estimated for the whole windtunnel, is on average at 2.5%.

Figure 4.4.8: Turbulence intensities for the empty channel with flow, 16 px
5 Experimental Results

In the following chapter results from the PIV-measurements are presented and discussed. Overviews as well as detailed plots of the measured planes for the Pyramid-tree, large branch of the spherical tree and small parts of both trees are presented and discussed. The coordinate origin is, as defined in figure 4.1.1, in the middle of the trunk on the tunnel bottom.

5.1 Pyramid-tree

For the pyramid tree, see table 3.2 for its dimensions, measurements were carried out at two different x-y-positions. These are located at the same x-y-position as the validation planes but have varying heights z. For the position further downstream 32 planes are recorded, from 6 – 16 cm with an increment of 2 cm, from 16 – 40 cm with 1 cm increments and from 40 – 44 cm again with a 2 cm distance. For the position closer to the trees 29 heights are measured, the lowest plane at 12 cm, while the following planes and corresponding increments stay the same. For purposes of presentation the plane closer to the tree is mirrored that both planes can be shown on the same side. In order to ensure convergence of the averaged quantities, such as velocity magnitude or turbulence intensity, 300 vector fields are calculated for each plane and height.

An overview of the mean velocity fields is given in figure 5.1.1. The imprint is visible in the planes further downstream in figures 5.1.1 and 5.1.2. The velocity magnitude defect behind the tree leads to velocities slower than 1 m/s, while outside of the tree, mostly in the corners of the tunnel, an acceleration of the flow up to 3.2 m/s is observed (see section 4.1.1). Figures 5.1.3 - 5.1.6 show instantaneous velocity fields at z = 14, 24, 29 and 34 cm. The dark gray parts are at the same corresponding height as the recorded data, while the transparent parts are displayed for orientation and clarity.
Figure 5.1.1: Overview of the PIV measurements for the Pyramid-tree, each plane shows the average velocity magnitude

Figure 5.1.2: Detailed view in flow direction, tree opacity 30%
Figure 5.1.3: Instantaneous velocity field at $z = 14$ cm
Figure 5.1.4: Instantaneous velocity field at $z = 24\, \text{cm}$
Figure 5.1.5: Instantaneous velocity field at $z = 29$ cm
Figure 5.1.6: Instantaneous velocity field at $z = 34\,\text{cm}$
The turbulence intensity, again in an overview, is plotted in figure 5.1.7. It reaches its maximum of around 17% in the wake of small branches in the tree’s lower part which only have the trunk upstream of them. Figure 5.1.8 shows the results from another direction. It is obvious that \( T_u \) exceeds 10% only in distinct areas close behind the tree and further downstream in the middle of the lower part of the tree where fewer branches are located and the wake behind the trunk dominates, it seems that the branches reduce \( T_u \). This matches the findings from Cafiero et al. [3] very well, who suggest that the largest part of modal energy is associated with vortex shedding at the largest structures. The turbulence decay seems to happen faster for turbulence generated by small branches than from the comparatively wide trunk and the few small branches. Furthermore, it is worth noticing that in 5.1.8 it can be seen that the ongoing decay lowered \( T_u \) down to values of around 7% only 50 cm downstream of the tree. Additionally, \( T_u \) is interpolated from planar data onto a volumetric dataset of the resolution 255 x 255 x 190 grid points, which is used to create contour plots, which can be seen in figure 5.1.9 for \( T_u = 7.5, 10, 12.5 \) and 15%.

Figure 5.1.7: Overview of the measured turbulence intensity for the Pyramid-tree
Figure 5.1.8: Turbulence intensity behind the Pyramid-tree

Figure 5.1.9: Contours of $Tu = 7.5, 10, 12.5, 15\%$ from the interpolated data
Figures 5.1.10 and show 5.1.11 the contour plots again, now with an emphasis on the areas of higher turbulence behind individual branches, which can be easily spotted. Note also the high value of $T_u$ behind the trunk.

**Figure 5.1.10:** Areas of higher $T_u$ behind distinct (clusters of) branches, $T_u = 7.5, 10, 12.5, 15\%$

**Figure 5.1.11:** Top view on the contour-plot, $T_u = 7.5, 10, 12.5, 15\%$
5.2 Large Branch of the spherical tree

For the branch of the spherical tree, as presented in figure 3.2.10, 600 double-images are taken and analyzed in order to further improve convergence of the averaged quantities. Again, overview representations for the velocity magnitude and $T_u$ are presented. The velocity magnitude is displayed in figure 5.2.1, $T_u$ in figure 5.2.2. For each x-y-position 11 planes are measured. Both areas have dimensions of 7.9 cm x 7.9 cm, while each of the 255 x 255 calculated vectors now describes an area of 0.31 mm x 0.31 mm. This holds for all further PIV-analyses in this work. The centers of the planes are located at (11.7 cm, 3.5 cm, $z$) and (21.2 cm, 3.5 cm, $z$). The bottom plane is located at a height of $z = 9.3$ cm, which is the lowest point of the small branches. From there all further planes are measured with an increment of 1 cm each. Therefore the top plane is at $z = 19.3$ cm. Even though not visible in the presented overview image, the imprint, as mentioned in [18], is clearly visible in form of the velocity defect in the wake.

![Figure 5.2.1: Overview of the average velocity for the large spherical branch](image-url)
For a more detailed study of the development of $Tu$ three planes at heights $z = 10.3\,\text{cm}$, $13.3\,\text{cm}$ and $16.3\,\text{cm}$ are selected, which are displayed in figures 5.2.3 and 5.2.4. Note that the lowest number for $Tu$ here is 5%. 

Figure 5.2.2: Overview of the turbulence intensity for the large spherical branch

Figure 5.2.3: Locations of the selected planes
Experimental Results

(a) Top view on planes at $z = 10.3\, \text{cm}$

(b) Top view on planes at $z = 13.3\, \text{cm}$

(c) Top view on planes at $z = 16.3\, \text{cm}$

Figure 5.2.4: The selected planes for a more detailed study of $T_u$
From the overview image 5.2.2 it is obvious that $Tu$ exceeds 10% only very closely behind the branches and then rapidly decreases. This observation very well matches the conclusions from the Pyramid-tree, which shows similar behavior.

Figure 5.2.4a shows that the highest $Tu$ is up to 25% at 8 cm behind the trunk. Furthermore, at 20 cm values of 15% can still be found. Further downstream at 25 cm from the trunk, at the second plane’s downstream-end, the wakes of small branches have merged with the trunk’s larger wake and hence $Tu$ exceeds 10% over almost the whole width of the tree. From 13% in the middle of the tree $Tu$ decreases down to about 10% at 6 cm in $y$-direction. The near wake of the smaller branches, visible in the top half of the first plane, reaches values of $Tu$ of up to 18%, which then decreases very fast to values of $Tu < 10\%$ only ca. 5 cm further downstream.

A more drastic and even faster decreasing distribution of $Tu$ is observed in the planes 3 cm higher at $z = 13.3$ cm, presented in figure 5.2.4b. Here, the plane closer to the tree only has values of $8.5\% < Tu < 10\%$. Further downstream, in the second plane, besides the wake close to the top corner and the one just below the middle of the picture, most of the turbulence ceases to exceed even a value of 8.5%.

For the planes another 3 cm higher, then at $z = 16.3$ cm, the near wake just behind the cluster of small branches (compare figure 5.2.4) $Tu$ again reaches up to 19%, which then decreases swiftly until an estimated value of 10% in between the measured planes and values of around 5% at the downstream end of the measured plane. Again, merging of the large central wake and the smaller one from further outside is visible.

Figures 5.2.5 and 5.2.5 show contour plots of $Tu$ from the volumetric dataset obtained from interpolation between the measured planes. They suggest that the top part of the turbulence generator only has a significant impact in the near wake. The lower part, especially the trunk, generates the most turbulence, while the large number of small structures seems to reduce $Tu$. 

Figure 5.2.5: Contourplot of $Tu$ from interpolated dataset, $Tu = 5, 10, 15, 20, 25\%$

Figure 5.2.6: Visualization of high $Tu$ behind the trunk and close to the tree
5.3 Small Branch of the spherical tree

The smallest printed part of the spherical tree, displayed in figure 3.2.11b, is again analyzed using 600 double-images for good convergence of the statistics. Like for the large branch, the analyzed planes are 7.9 cm x 7.9 cm, large with their center being located at (6.8 cm, −0.5 cm, z). 11 planes are measured at heights of \( z = 1.5 \text{ cm}, 2.5 \text{ cm} \) and from there in 5 mm increments up to a maximum height of 7 cm. Overview images of both velocity magnitude and \( T_u \) can be seen in figures 5.3.1 and 5.3.2.

![Figure 5.3.1: Overview of the average velocity for the small spherical tree](image)

![Figure 5.3.2: Overview of the turbulence intensity for the small spherical tree](image)
In order to get a more detailed idea about the turbulence intensity the interpolated data is used again. It is presented from three different points of view in figure 5.3.3. It becomes clear that $Tu$ decays over a short distance.

![Figure 5.3.3: Three representations of the isosurfaces for $Tu = 5, 10, 15, 20\%$ for the small spherical tree](image)

At lower heights, with the trunk as the only obstacle, the initial turbulence is the highest (compare also figure 5.3.2) and decays to values lower than 15\% at $x = 100$ mm and even below 10\% for the upper part. In the middle part, $Tu$ already drops under 15\% at $x = 50$ mm. In contrast to the trunk and more central parts of the tree, in the top part only small
branches of the third and fourth generation generate turbulence. Decay there happens is even faster, $Tu$ falls below 10% at already $x = 60$ mm. Note that the turbulence is generated by structures being closer to the measured areas than it is the case for lower parts of the tree.

It is a striking, yet not desired, observation that behind the very center of the tree (at $z = 35$ cm, where the trunk ends and the next generation emerges) very fast falls below 10%. This particular behavior is shown in figure 5.3.4. A possible reason for this might be that upstream of the region with low $Tu$ there is a branch of the second generation in direction of the flow (compare figure 5.3.3, bottom left image). However, the maximum values for $Tu$ are located behind the trunk and, with faster decay, right after the small clusters of branches.

Figure 5.3.4: View of $Tu$ (note the low values numbers in the very middle) behind the trunk's tip
For comparison to the numerical data, the vorticity is calculated and plotted in figure 5.3.5. Note that from the measured data in the $x$-$y$-plane only the $z$-component can be obtained.

*Figure 5.3.5: Z-component of the time-averaged vorticity for the small spherical tree*
5.4 Small Branch of the Pyramid-tree

The smallest part of the Pyramid-tree is shown in figure 3.2.11a. Measurements for this part are again done by recording 600 double-images. Like for the previous analyses, the planes have dimensions of 7.9 cm x 7.9 cm large with centers at (9.1 cm, 3.1 cm, z). 16 planes are measured, the lowest being at z = 1 cm. From there the other planes are measured in 5 mm increments up to a maximum height of 8.5 cm. The velocity magnitude and $Tu$ are presented in figures 5.4.1 and 5.4.2.

Similar to previous observations, the largest values for $Tu$ of more than 20 % are found in the wake of the trunk. Even only a little higher, where small branches in addition to the trunk generate turbulence, $Tu$ does not exceed 15 % anymore.

*Figure 5.4.1: Overview of the average velocity for the small Pyramid-tree*
In figure 5.4.3 the same pattern as found for the small *spherical* tree is observed again. The highest $Tu$ is found in the wake of the trunk with values of around 20% at $x = 50$ mm, where the measurement area starts, and 15% until $x = 100$ mm.

Right in the middle of the tree, where the new branches emerge from the trunk, again low $Tu$ is found, as well as in the downstream wakes of middle of the outer and upper cluster of branches (see figure 5.4.3 on the bottom left). Again, the treetop seems to be far less effective in generating turbulence than the middle and lower part. $Tu$ is lower than 10%. 

![Figure 5.4.2: Overview of the turbulence intensity for the small Pyramid-tree](imageurl)
Figure 5.4.3: Three representations of the iso-surfaces for $T_u = 5, 10, 15 \text{m} 20\%$ for the small Pyramid-tree.
Like for the small *spherical* tree, for comparison to the numerical data, the vorticity is derived and plotted in figure 5.4.4. Again, to the 2D-data only allows the $z$-component to be calculated, which in the present case was done by calculating the curl of the averaged velocity.

*Figure 5.4.4: Z-component of the time-averaged vorticity for the small Pyramid-tree*
Experimental Results

5.5 Young Spruce Tree

In order to compare the designed trees additional measurements are conducted with a small spruce tree, figure 5.5.1. Its trunk is about 1 cm in diameter, the canopy is ca. 25 cm wide and deep, the total height is 40 cm. Since it is mounted below the tunnel, its effective height is only 35 cm. At positions (21.3 cm, -3.3 cm, z) and (30.9 cm, -3.3 cm, z) 7 planes (z = 13 – 25 cm, 2 cm increments) are measured. In contrast to the 3D-printed rigid trees the spruce begins to bend and oscillate with small amplitudes of around 2 cm at the top in the airflow.

![Image of spruce tree](image1.png)

Figure 5.5.1: The spruce tree for correlations with the printed models

Representations of the velocity magnitude and Tu can be found in figures 5.5.2 and 5.5.3. The Pyramid-tree (see sec. 5.1) has about the same height (44 cm), but is 20 cm wider (44 cm). The smallest velocity magnitude for both is at 0.5-0.6 m/s. It is worth noting that the overall velocity defect and Tu match well. For the Pyramid-tree the maximum Tu is 18 %, while being 15 % for the spruce. At ca. x = 35 cm Tu falls below 10 % for both cases almost
in every plane. The spruce’s imprint is not clearly visible because it does not have a distinct, strictly defined pattern of branches. The significant differences between the measured planes in figure 5.5.3 could indicate that the number of samples (600) is too low.

Figure 5.5.2: Overview of the average velocity magnitude for the spruce

Figure 5.5.3: Overview of the turbulence intensity for the spruce
5.6 Parts of a yew tree

The printed trees are perfectly rigid for a wind speed of 2.8 m/s, while the spruce was only lightly affected, as previously stated. Therefore, in order to further investigate the influence of flexibility, small yew parts are used (figure 5.6.1). All experiments with the yews were conducted shortly after cutting, therefore the assumption of constant mechanical properties compared to the living yew seems reasonable. The two Y-shaped branches are each about 13 cm high and ca. 7 cm wide at the very top. With the tunnel running, the airflow bends them by 30° as it can be seen in figure 5.6.2. Additionally, two of the branches are placed into the wind-tunnel with a distance of 7 cm in flow direction in order to obtain information about the interaction of their wakes. They are presented in figure 5.6.3. The experimental data was obtained from two planes with the same dimensions as used previously (7.9 cm x 7.9 cm) with centers at (9 cm, 3 cm, z) and (19 cm, 3 cm, z), while this time only two heights are measured, \( z = 5 \) and 8.5 cm. For the two branches one after another, only the second, further downstream, plane at \( z = 8.5 \) cm is measured.

Figure 5.6.1: The yew branch without (left) and with (right) flow, view from the top
Figure 5.6.2: The yew branch bent by 30°, flow from the left

Figure 5.6.3: The two yew branches, views from the side and top
Figures 5.6.4 and 5.6.5 show the measured planes, as well with an approximate location of the branch, of both velocity magnitude and $T_u$. The range of velocity is $5.5 \times 10^{-3} - 3.17$ m/s, while for $T_u$ it spans from 2.68 % to 29.4 %. In contrast to previously discussed data the mean velocity average is at about zero in the wake at $z = 8.5$ cm, while just outside the wakes it almost hits 3.2 m/s. The shear layer is clearly visible, here $T_u$ attains its maximum. $T_u$ is reaches values of >25 % in the shear-layers right behind the branches, the most significant plane, again the one at $z = 8.5$ cm, is displayed in figure 5.6.6. Figure 5.6.7 shows the downstream decay at the same height with an adjusted legend for clarification and further comparison.

Figure 5.6.4: Overview of the average velocity magnitude for the single yew branch
Figure 5.6.5: Overview of the turbulence intensity for the single yew branch

Figure 5.6.6: Turbulence intensity for the single yew branch at $z = 8.5\, \text{cm}$
The results for the two yew branches with 7 cm stream-wise distance are shown in figures 5.6.8 - 5.6.11. The velocity span is 0.44 - 2.98 m/s, while for $Tu$ it is between 2.72 and 19.9 %. Comparison of figures 5.6.7 and 5.6.11, which are measured at the same height and distance behind the (last) branch, show $Tu$ of >10 % for the two branches almost everywhere in the wake, while for only one branch, it mostly drops under 10 %. Hence, the two branches in line significantly increase $Tu$ in the wake and slow down the decay.
Figure 5.6.8: Overview of the average velocity magnitude for the two branches

Figure 5.6.9: Overview of the turbulence intensity for the two branches
Figure 5.6.10: Velocity magnitude for the two branches

Figure 5.6.11: Turbulence Intensity for the two branches
6 Numerical Simulations

This chapter is intended to discuss the numerical simulations and compare and correlate them to the experimental data. All simulations are conducted using FLUSI, the fluid-structure-interaction solver which has been developed by Thomas Engels. It is designed to run on massively parallel supercomputers and makes use of the volume penalization method with an effective Fourier-discretization for the fluid. More detailed information can be found in [8]. Simulations are conducted for the smallest printed parts of both the spherical and Pyramid-tree (figure 3.2.11). Their experimental study is discussed in the previous chapter in sections 5.3 and 5.4.

Slices at the same heights as used in the experiments are extracted from the numerical results for the purpose of direct comparison. Each slice covers an area of 15.6 cm x 7.8 cm with a resolution of 2048 x 1024 pixels. Hence, each data point in the extracted sheets represents an area of 0.076 mm x 0.076 mm.

As mentioned in section 5.2, the PIV-sheets with a 255 x 255 pixels resolution cover an area of 0.31 mm x 0.31 mm each. The numerical data offers a 16 times better resolution compared to the respective PIV data (4.09 times in each direction, hence actually the resolution is 16.63 times higher). At the same time, the covered area of the is almost twice as large (1.949 times) as the area covered by PIV-data.
6.1 Spherical Tree - Simulation & Experiment

Planes are extracted at $z = 25, 40, 45, 50$ and $55$ cm, as it can be seen in figures 6.1.1 and 6.1.2.

Figure 6.1.1: Location of the compared planes, side view

Figure 6.1.2: Location of the compared planes, isometric view
First, the influence of the missing $z$-component for the comparison with the experimental data is investigated. For this purpose, the mean velocity (figure 6.1.3) is calculated from the $x$- and $y$-components of the velocity (left) and also in all three dimensions (right). No visible difference between the two- and three-dimensional calculations is found. This justifies comparison of the 2D-PIV data to the 3D numerical data in the following analysis. The periodic boundary conditions of the simulation are visible. In the center the inflow velocity is found to be at around 2.3 m/s, while further along the $y$-axis it increases up to 3 m/s, which has to be kept in mind for comparison.

Figure 6.1.3: Comparison between the 2D- and 3D average velocity magnitude, $z = 25\,\text{cm}$
Figures 6.1.4 - 6.1.23 show comparisons of instantaneous and mean velocity fields, $T_u$, as well as the vorticity for each of the five selected planes. Even though $T_u$ is calculated from 2D-velocity-vectors for the experimental data and by using the 3D-data from the simulations, the agreement is remarkable. Experimental results are superimposed with the numerical results on the left side, while only numerical data is shown on the right side. Note that for the vorticity, only the $z$-component can be obtained from a velocity vector in the $x$-$y$-plane. Note the noise in the instantaneous velocity fields, which is not visible in the averaged fields anymore. The overall agreement between experimental and numerical data is excellent, even though the significantly lower resolution of the PIV-data is visible.

For the vorticity, the same comparisons are made, while the scale of the numerical data is set to span the range covered by the experimental results. For the comparison between the $z$-component and the vorticity magnitude, the ranges of the colormaps are also adjusted for better visibility of the results. Extremely high maximum and minimum values of both the $z$-component (around $\pm 1.5 \cdot 10^4$ $1/s$) and the magnitude (around $2.5 \cdot 10^4$ $1/s$) very close and directly at the branches would otherwise make visual comparison impossible.
Figure 6.1.4: Comparison of two instantaneous velocity fields, $z = 25 \text{ mm}$
Figure 6.1.5: Comparison of the average velocity, $z = 25 \text{ mm}$
Figure 6.1.6: Comparison of $Tu, z = 25 \text{ mm}$
Figure 6.1.7: Comparison of the average vorticity (z-component), $z = 25\, mm$
Figure 6.1.8: Comparison of two instantaneous velocity fields, $z = 40\ mm$
Figure 6.1.9: Comparison of the average velocity, \( z = 40 \text{ mm} \)
Figure 6.1.10: Comparison of $Tu$, $z = 40 \text{ mm}$
Figure 6.1.11: Comparison of the average vorticity (z-component), $z = 40\, \text{mm}$
Figure 6.1.12: Comparison of two instantaneous velocity fields, $z = 45\ mm$
Figure 6.1.13: Comparison of the average velocity, $z = 45 \text{ mm}$
Figure 6.1.14: Comparison of $Tu$, $z = 45 \text{mm}$
Figure 6.1.15: Comparison of the average vorticity (z-component), $z = 45\, \text{mm}$
Figure 6.1.16: Comparison of two instantaneous velocity fields, z = 50 mm
Figure 6.1.17: Comparison of the average velocity, $z = 50\ mm$
Figure 6.1.18: Comparison of $Tu$, $z = 50 \, mm$
Figure 6.1.19: Comparison of the average vorticity (z-component), $z = 50 \text{ mm}$
Figure 6.1.20: Comparison of two instantaneous velocity fields, $z = 55 \text{ mm}$
Figure 6.1.21: Comparison of the average velocity, \( z = 55 \text{ mm} \)
Figure 6.1.22: Comparison of $Tu, z = 55 \text{ mm}$
Figure 6.1.23: Comparison of the average vorticity (z-component), $z = 55\, mm$
6.2 Pyramid-tree - Simulation & Experiment

Planes are extracted at \( z = 10, 25, 35, 50, 65 \) and 75 cm. The location planes is presented in figures 6.2.1 and 6.2.2.

*Figure 6.2.1: Location of the compared planes, side view*

*Figure 6.2.2: Location of the compared planes, isometric view*
Figures 6.2.3 - 6.2.26 display comparisons of instantaneous and mean velocity fields, $Tu$ and vorticity for the six selected planes. $Tu$ is calculated from the 2D velocity vectors for the experimental planes, while 3D-data from the simulation is used for calculations. Again, only the $z$-component of the vorticity can be obtained from velocity vectors in the $x$-$y$-plane. The simulation show wider wakes as visible in 6.2.4 or 6.2.20. Further downstream, the discrepancy between simulation and experiment closer to the tree diminishes. The maximum values of the vorticity $z$-component are $\pm 5.38 \cdot 10^3$ $1/s$ and for the magnitude the maximum is $7.5 \cdot 10^5$ $1/s$. 
Figure 6.2.3: Comparison of two instantaneous velocity fields, $z = 10\, \text{mm}$
Figure 6.2.4: Comparison of the average velocity, $z = 10 \text{ mm}$
Figure 6.2.5: Comparison of $Tu$, $z = 10\ mm$
Figure 6.2.6: Comparison of the average vorticity (z-component), $z = 10 \text{ mm}$
Figure 6.2.7: Comparison of two instantaneous velocity fields, $z = 25\ mm$
Figure 6.2.8: Comparison of the average velocity, $z = 25\, mm$
Figure 6.2.9: Comparison of $Tu$, $z = 25$ mm
Figure 6.2.10: Comparison of the average vorticity (z-component), $z = 25 \text{ mm}$
Figure 6.2.11: Comparison of two instantaneous velocity fields, $z = 35\,\text{mm}$
Figure 6.2.12: Comparison of the average velocity, $z = 35 \text{ mm}$
Figure 6.2.13: Comparison of $Tu, z = 35 \text{ mm}$
Figure 6.2.14: Comparison of the average vorticity (z-component), $z = 35 \text{ mm}$
Figure 6.2.15: Comparison of two instantaneous velocity fields, $z = 50 \text{ mm}$
Figure 6.2.16: Comparison of the average velocity, $z = 50$ mm
Figure 6.2.17: Comparison of $Tu, z = 50 \text{ mm}$
Figure 6.2.18: Comparison of the average vorticity (z-component), $z = 50 \text{ mm}$
Figure 6.2.19: Comparison of two instantaneous velocity fields, $z = 65 \text{ mm}$
Figure 6.2.20: Comparison of the average velocity, $z = 65$ mm
Figure 6.2.21: Comparison of $T_u, z = 65\, \text{mm}$
Figure 6.2.22: Comparison of the average vorticity (z-component), \( z_{65} = \text{mm} \)
Figure 6.2.23: Comparison of two instantaneous velocity fields, $z = 75\, \text{mm}$
Figure 6.2.24: Comparison of the average velocity, $z = 75\ mm$
Figure 6.2.25: Comparison of $Tu$, $z = 75 \text{ mm}$
Figure 6.2.26: Comparison of the average vorticity (z-component), $z = 75\ mm$
7 Conclusion & Outlook

The present thesis experimentally and numerically investigates turbulence generators inspired by fractal trees. Different trees are designed, 3D-printed in parts, assembled and used for PIV-investigations in a small wind-tunnel. Furthermore, numerical simulations of the same trees are conducted and the respective results are compared. Very good agreement between experiment and simulation of instantaneous and averaged velocity fields, as well as turbulence intensity is found. For the vorticity it becomes clear that the spatial resolution of the PIV-system is not high enough in order to capture the high vorticity in the near wake of individual branches. Real plant foliage, in particular a small spruce tree, as well as branches of a yew tree, is experimentally investigated. The results for the spruce indicate similar behavior compared to the printed parts. Larger differences are found for the yews, which in the wind-tunnel bend up to 30°. Two of them in line significantly increase the turbulence intensity and slow down its decay. A dominant observation for turbulence generated by fractal grids, according to literature (see chapter 2), is the increase turbulence after the grid until a certain distance and its decay from there. Since the structures investigated in the present paper are three-dimensional there is no such point. A more sophisticated approach including the three-dimensionality would be desirable in order to find systematic similarities between the different 3D-trees. In future works, basing on the present work, in order to obtain more information about the flow field, it would be desirable to measure time-resolved drag forces of the tree, preferably with a load cell. Furthermore, stereo-PIV could be employed in order to measure the z-component of the velocity. With that information, vertical fluxes and structures could be investigated. Furthermore, interpolation of volumetric data could be performed more reliably and would not be limited to scalar fields, as it was the case in this work. Time-resolved PIV or hot-wire anemometry could be used for time-resolved measurements.
The *H-tree* was constructed but not used for the experiments in this work. One inherent advantage of its appearance is that is does not have an upper part with less branches generating turbulence. The hypothesis of a larger wake and increased turbulence towards the centerline then could be examined and discussed.

The influence of the Reynolds-number onto turbulence generated by 3D-fractal structures remains unclear at this point. Therefore repeating the measurements for different wind speeds could give insights into the Reynolds-dependence. In order to keep the efforts to be made within reasonable limits, the measurement process, including the moving of the trees (or the laser-camera unit) could be automated. For this purpose, *LabVIEW* in combination with a linear motor seems to be a good basis to start from.

For a more extensive study, an array of trees could be investigated. Furthermore, in an attempt to exploit the strong imprint of the fractal geometry in the flow patterns, investigations concerning flow control mechanisms could be made.
Appendices
function my_own_tree_example

clear all
clc

% This script can generate 3D trees with an arbitrary number of branches per generation. Outputs the coordinates of the start/end point and the length for each cylinder to an ascii file. This file can be read with FLUSI.

%%% settings generations
% rho – variation of the length between generations.
% rho < 1: branches are getting shorter
rho = 2^(-1/3);

% number of max generations, min, max diameters
Jmax = 9;
d_max = 0.08;
d_min = 0.006;

% length/diameter ratio
delta_gen = (d_min/d_max)^(1/(Jmax-1));

%%% settings trunk, not fractal yet
% length, usually unity
L0 = 1;
L_t = 1*L0;
% trunk diameter
d0 = d_max;
% position of trunk (0 0 0 by convention)
x0=[0; 0; 0];

%%% settings angles
% angles for each branch, in radians
% length(alpha) determines number of branches per
\begin{verbatim}
generation

alpha = [90 90]*pi/180;
beta = [0 180]*pi/180;
gamma = [90 90]*pi/180;

%%%% generation of tree
generation
figure(1); clf
% draw the trunk
line( [x0(1),x0(1)] , [x0(2),x0(2)] , [x0(3),x0(3)+L_t]);

fid = fopen( 'tree_data.in', 'w');
fprintf(fid, '%e %e %e %e %e %e
', x0, x0+[0;0;L_t], d0/2);

% the first generation is not rotated, so its
transformation matrix is a unit matrix
M = eye(3);

% new base vector is tip of trunk
x_base = [x0(1); x0(2); x0(3)+L_t];

% we start at the first level (zeroth is the trunk)
j =1;

% recursively build tree
draw_branch(j , Jmax , rho , L0/2, d_max , delta_gen , M, x_base , alpha
, beta , gamma , fid);

fclose(fid);
axis equal
view(45,26)
grid on

end

function draw_branch(j , Jmax , rho , L , d, delta , M1, x_base,
alpha , beta , gamma , fid )
\end{verbatim}
% recursive drawing of a fractal tree
% j – level counter (used to abort when we’re done)
% rho – ratio of lengths (between generations, <1 means shrinking)
% L – length of my branches in this generation
% MI – transformation matrix used in the OLD generation. used to rotate relative to the previous branch
% x_base – base vector of my branches (=tip of last generation)

for i = 1:length(alpha)
    % MI is rotation matrix from the preceeding generation , to which we add, now, the RELATIVE rotation:
    M = Rx(gamma(i))*Ry(alpha(i))*Rz(beta(i))*MI;

    % in the rotated coordinate system, the tip is at
    x_tip_rel = [0; 0; L];

    % applying the inverted rotation matrix fetches this vector in the system of the preceeding generation. NOTE: applying inv(M) brings us back to the global system
    x_tip = x_base + transpose(M)*x_tip_rel;

    % draw line
    line([x_base(1),x_tip(1)], [x_base(2),x_tip(2)], [x_base(3),x_tip(3)]);

    fprintf(fid,'%e %e %e %e %e %e
',x_base,x_tip,d/2);

    % recursion
    if (j<Jmax)
        draw_branch(j+1,Jmax,rho,L*rho,d*delta,delta,M,
                    x_tip, alpha, beta, gamma, fid)
    end
end
function Rx=Rx(angle)
    % rotation matrix around x axis
    Rx=[1 0 0; 0 cos(angle) sin(angle); 0 -sin(angle) cos(angle)];
end

function Ry=Ry(angle)
    % rotation matrix around y axis
    Ry=[cos(angle) 0 -sin(angle); 0 1 0; +sin(angle) 0 cos(angle)];
end

function Rz=Rz(angle)
    % rotation matrix around z axis
    Rz=[cos(angle) +sin(angle) 0; -sin(angle) cos(angle) 0; 0 0 1];
end
Example of the synchronizer settings
Example of some PIVLab settings
IMAGES & VECTORFIELDS

Example image 1
Velocity vectors from example image 1 (and its corresponding second one)
Example image 2

Velocity vectors from example image 2 (and its corresponding second one)


